Table 1 Solution of the differential equation

$\alpha^4 - \beta^4 > 0$	$\alpha^4 = \beta^4$	$\alpha^4 - \beta^4 < 0$
$w = \dfrac{BM_0 + Q_0}{D(B - A)(4\alpha^2 - A^2)} e^{-Ax} + \dfrac{AM_0 + Q_0}{D(B - A)(B^2 - 4\alpha^2)} e^{-Bx}$	$w = \left[\dfrac{Q_0}{Dm\alpha^2} - \dfrac{M_0}{2D\alpha^2}\right]e^{mx} + \left[\dfrac{Q_0}{2D\alpha^2} - \dfrac{M_0}{Dm}\right]xe^{mx}$	$w = \dfrac{e^{-Rx}}{2D\beta^4}\left[\left(-\beta^2 \cos Px + \dfrac{R}{P}\beta^2 \sin Px\right)M_0 + \left(\dfrac{\alpha^2}{P} \sin Px - R \cos Px\right)Q_0\right]$
$M_x = \dfrac{Q_0[e^{-Ax} - e^{-Bx}]}{B - A} + \dfrac{M_0[Be^{-Ax} - Ae^{-Bx}]}{B - A}$	$M_x = e^{mx}[M_0 + (Q_0 - mM_0)x]$	$M_x = \dfrac{e^{-Rx}}{P}[Q_0 \sin Px + M_0(P \cos Px + R \sin Px)]$
$Q_x = \dfrac{Q_0[-Ae^{-Ax} + Be^{-Bx}]}{B - A} + \dfrac{M_0[-ABe^{-Ax} + ABe^{-Bx}]}{B - A}$	$Q_x = e^{mx}[Q_0 + (mQ_0 - 2\alpha^2 M_0)x]$	$Q_x = \dfrac{e^{-Rx}}{P}[Q_0(P \cos Px - R \sin Px) - M_0(R^2 + P^2) \sin Px]$
where	where	where
$A = [2\alpha^2 + 2(\alpha^4 - \beta^4)^{1/2}]^{1/2}$	$m = -(2)^{1/2}\alpha$	$R = (\beta^2 + \alpha^2)^{1/2}$
$B = [2\alpha^2 - 2(\alpha^4 - \beta^4)^{1/2}]^{1/2}$		$P = (\beta^2 - \alpha^2)^{1/2}$

Table 2 Internal forces at joint

G_c, psi	Q_1, lb/in	Q_2, lb/in	Q_3, lb/in	M_1, in lb/in	M_2 in -lb/in	M_3 in -lb/in
10^3	82 6	17 3	−100 0	−132 9	−3 1	204 0
10^4	143 1	20 3	−163 4	−152 2	−4 4	183 4
10^5	158 9	20 8	−180 0	−156 5	−4 6	178 9

By combining Eqs (1–6), the following differential equation results:

$$\left[1 + \frac{N_0}{D_{Qx}}\right]\frac{d^4w}{dx^4} - \left[\frac{E_y}{D_{Qx}r^2} + \frac{1 - \mu_x\mu_y}{D_x}N_0\right]\frac{d^2w}{dx^2} + \frac{E_y(1 - \mu_x\mu_y)}{D_x r^2} w = \frac{1 - \mu_x\mu_y}{D_x}\left[\frac{\mu_y}{r} N_0 + p\right] \quad (7)$$

Solution of the Differential Equation

Equation (7) was solved for the case of a long cylinder subjected to a moment M_0 and a transverse shear Q_0, as shown in Fig 1 The normal pressure p and the axial force N_0 were made equal to zero The solution for Eq (7) is shown in Table 1 The constants are defined as follows:

$$\alpha^2 = \frac{E_y}{4D_{Qx}r^2} \qquad \beta^4 = \frac{E_y(1 - \mu_x\mu_y)}{4D_x r^2} \qquad D = \frac{D_x}{(1 - \mu_x\mu_y)}$$

The influence coefficients for the displacement and rotation of the edge of the shell are

$$[w]_{x=0} = -\left[\frac{(\beta^2 + \alpha^2)^{1/2}}{2\beta^4 D}\right]Q_0 - \left[\frac{1}{2\beta^2 D}\right]M_0 \quad (8)$$

$$\left[\frac{dw}{dx}\right]_{x=0} = \left[\frac{2\alpha^2 + \beta^2}{2D\beta^4}\right]Q_0 + \left[\frac{(\alpha^2 + \beta^2)^{1/2}}{D\beta^2}\right]M_0 \quad (9)$$

Equations (8) and (9) are valid for $\alpha^4 - \beta^4 > 0$, for $\alpha^4 = \beta^4$, and for $\alpha^4 - \beta^4 < 0$

Example

The joint of a sandwich pressure vessel with various shear rigidities is analyzed here to show the effects of shear deflections on the internal forces Figure 2 illustrates the pressure vessel and idealized structural model In analyzing the joint, the following material properties were used: elastic modulus equals 3 5 $\times 10^6$ psi in the x direction and 4 5 $\times 10^6$ psi in the y direction; Poisson's ratio equals 0 15 in both directions The internal pressure is 200 psi The internal forces Q_1, Q_2, Q_3, M_1, M_2, and M_3 were determined for each of three different sandwich core shear moduli G, and the results are shown in Table 2 It can be seen from Table 2 that shear deflections may have an important influence on the internal loads

The variation of the shear force Q_x as a function of x, plotted in Fig 3, is for section 3 of the joint for each value of core shear rigidity The variation of the moment M_x as a function of x is plotted in Fig 4 The shear forces Q_x and moments M_x were obtained from Table 1 It is obvious from Figs 3 and 4 that the shear and moment at the edge of a cylinder die out faster if the shear rigidity is large

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Deformations and Stresses in an Axially Restrained Beam

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Nomenclature

a_0 = amplitude of lateral deflection of unrestrained beam due to lateral load and thermal gradients

a_1 = amplitude of lateral deflection of restrained beam

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l = length of beam column
q = lateral load
w = lateral deflection of restrained beam = $a_1 f(x)$
x = axial distance along beam column
$f(x)$ = first fundamental lateral deflection mode
C = effective flexibility in axial direction = $(1/Kl) + (1/AE)$
EI, EA = bending and axial stiffness of cross section
F = restraining load
F_E = Euler load = $k\pi^2 EI/l^2$ [$k = 1$ (simple); $k = 4$ (clamped)]
K = stiffness of axial restraint
M_0 = moment in beam unrestrained in axial direction
$\bar{\alpha}\bar{T}$ = average axial strain of beam due to thermal expansion
$= \left(\frac{1}{l}\right) \int_l^0 \alpha T dx$

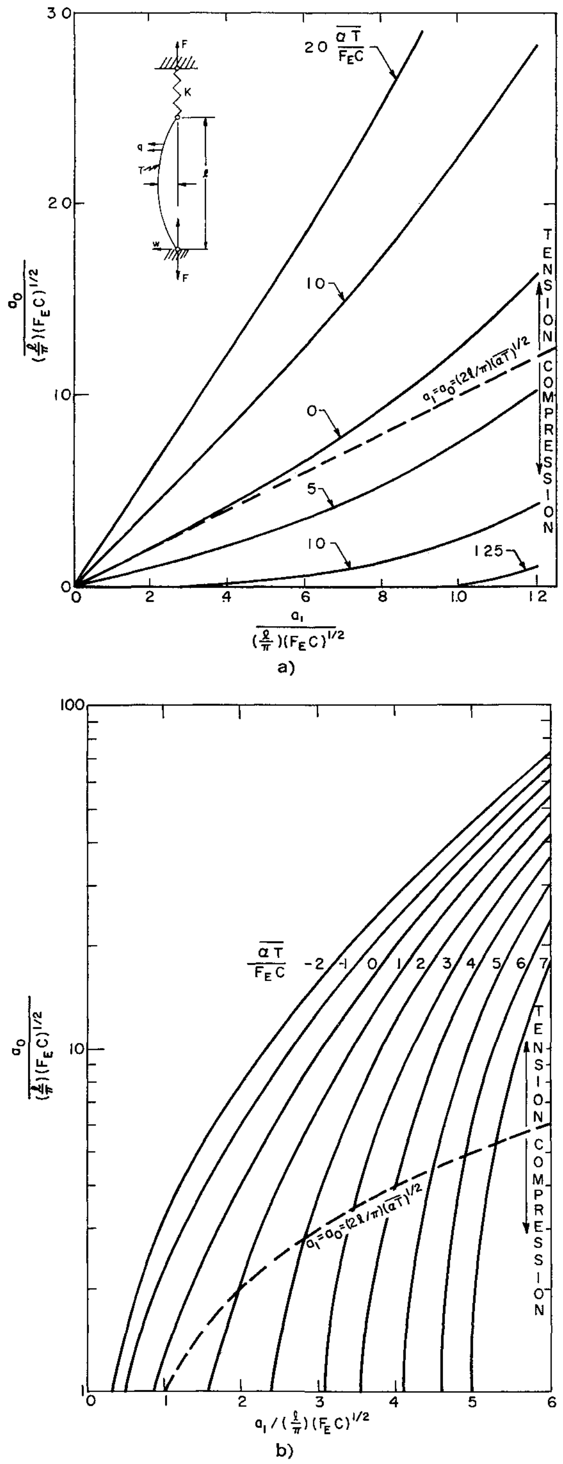

Fig 1 Unrestrained vs restrained lateral deflections of a beam column

An approximate solution [Eq (1)] is presented in Figs 1a and 1b for the deflection of a restrained beam column of arbitrary cross section and boundary conditions subjected to lateral loads and temperature The figures can also be employed to obtain the restraining axial load and the stresses The solution is based upon the assumption that the beam is of constant cross-sectional stiffnesses and that the lateral deflection can be approximated by the first fundamental mode as a column

The physical solutions of the nondimensional equation

$$a_1[1 - (\bar{\alpha}\bar{T}/F_E C)]/(l/\pi)(F_E C)^{1/2} + (\tfrac{1}{4})[a_1/(l/\pi)(F_E C)^{1/2}]^3 = a_0/(l/\pi)(F_E C)^{1/2} \quad (1)$$

exist in first quadrant Figure 1a presents small values of the final deflection parameter $[a_1/(l/\pi)(F_E C)^{1/2}]$ which differs only slightly from the unrestrained deflection parameter divided by $1 - (\bar{\alpha}\bar{T}/F_E C)$ This solution is similar to the "linear" classical beam-column formula which ignores membrane action and where the lateral deflection grows as the reciprocal of the difference of the critical to applied strain (or load) Figure 1b presents larger "nonlinear" solutions ($a_1 \rightarrow [4a_0 F_E C(l/\pi)^2]^{1/3}$) where the membrane action becomes significant

Lateral loads and negative values of the axial expansion parameter ($\bar{\alpha}\bar{T}/F_E C$) cause the restraining load F to be positive, resulting in final lateral deflections which are smaller than the unrestrained deflections ($a_0/a_1 > 1$) As the axial expansion parameter becomes increasingly positive, the restraining load will decrease and become negative (compression) The compressive loads will augment the unrestrained lateral deflections ($a_0/a_1 < 1$) As the expansion parameter is further increased, the lateral deflections and compressive load will continue to increase—the deflections at an increasing rate, but the compressive load at a decreasing rate The increasing compressive load will approach, but never attain, the column buckling load ($-F_E$)

The axial load and moment is obtained from the following equations:

$$F = [(a_0/a_1) - 1]F_E > -F_E \quad (2)$$

$$M = M_0 - Fw = M_0 - Fa_1 f(x) \quad (3)$$

Linearized Analysis of the Pressure Waves in a Tank Undergoing an Acceleration

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Introduction

To obtain insight into the effects of acceleration on fluid flows, the linearized equations for the one-dimensional flow in an accelerated closed tank of compressible fluid are solved for the acceleration prescribed as a known function of time The wave pattern is described in detail for the flow induced by an instantaneous constant acceleration beginning at time $t = 0$

Equations of Motion for a Gas under Acceleration of Its Container

In a fixed inertial system the equations for the one-dimensional isentropic flow of an ideal gas take the form

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